

# Notes on the Origin of Gravity and the Laws of Newton

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## Abstract

Following Verlinde's recent work on the origin of gravity and the laws of Newton, we consider further the origin of the starting point of Verlinde's work and Unruh effect. Simple derivations are given for this starting point and Unruh effect with the standard theory about quantum mechanics, special relativity and "vacuum" background based on the gauge field theory. Our studies give the physical origin of the Planck length. The present work supports further the idea that gravity is not a fundamental force. In the present work, the concept of holographic screen is not absolutely necessary to derive the universal gravity and the laws of Newton.

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## I. INTRODUCTION

The origin of gravity and the laws of Newton is a long standing issue. Even after the discovery of general relativity by Einstein, the answer about the origin of universal gravity is still far from satisfying because of two reasons: (i) there is still serious problem in the unification of quantum mechanics and general relativity; (ii) it is still not clear how to establish a unified theory of electromagnetic, weak, strong and gravitational forces. In a recent work, Verlinde [1] emphasized again the end of gravity as a fundamental force, which has already been suggested by Jacobson [2] almost fifteen years ago. A related idea was also proposed by Padmanabhan [3]. Verlinde's work has ignited intensive studies on the thermodynamical origin of gravity. These advances were motivated by the fact that the gravity law closely resembles the laws of thermodynamics and hydrodynamics [4–8].

Following Verlinde's work [1], we reconsider the assumption of the change of entropy  $\Delta S$  for an object having a displacement  $\Delta x$  and the Unruh effect which assuming a temperature for an accelerated object. This assumption and Unruh effect play an essential role in Verlinde's derivations on the universal gravity and the laws of Newton. In the present work, simple derivations are given for this assumption and Unruh effect with the standard theory about quantum mechanics, special relativity and “vacuum” background based on the gauge field theory. Our derivations establish further the logic basis of regarding the gravity as an entropic force. We also stress the local thermal equilibrium [2] in deriving the universal gravity. In addition to the dimensional consideration to get the Planck length, we try to argue that the Planck length is the coherence length of the fluctuating “vacuum” background in thermal equilibrium. As for the equivalence principle, the derivation of the Newton's law of gravity shows clearly that the gravitational mass is the same physical concept as the inertial mass. Therefore, general relativity is the natural generalization of the special relativity with the inclusion of gravity, where the weak equivalence principle is not an assumption any more.

## II. WHY IS THERE A CHANGE OF ENTROPY FOR AN OBJECT HAVING A DISPLACEMENT?

In the original work by Verlinde [1], the following postulation motivated by Bekenstein's work [4] about black holes and entropy, plays a key role in deriving the Newton's law of gravity

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \quad (1)$$

In the above formula,  $\Delta S$  is the change of entropy after a displacement  $\Delta x$  for a particle with mass  $m$ . Here we reconsider the physical mechanism of this postulation based on quantum mechanics, special relativity and “vacuum” background.

From the special relativity, we have

$$E^2 = p^2 c^2 + m^2 c^4. \quad (2)$$

From the quantum mechanics, the eigenfrequency is then

$$\omega = \frac{E}{\hbar} = \frac{\sqrt{p^2 c^2 + m^2 c^4}}{\hbar}. \quad (3)$$

If the space is regarded as a sort of fluctuating “vacuum” background due to the existence of various gauge fields, there is a strong coupling between the matter and the “vacuum” background. As a consequence, from the view of the “vacuum” background, we have the following coherence length for the particle:

$$l_c = \frac{2\pi c}{\omega}. \quad (4)$$

The factor  $c$  in the above equation is due to the fact that the gauge field in the “vacuum” has just the propagation velocity of  $c$ . If  $p^2/m \ll mc^2$ , we have

$$l_c \approx \frac{2\pi \hbar}{mc}. \quad (5)$$

When the coupling between the particle and the “vacuum” background is considered, the coherence length  $l_c$  gives the location resolution of the particle from the view of the “vacuum” background.

In Fig. 1(a), we show the motion of a particle along the dashed line. Because of the location resolution from the view of the “vacuum” background, the dashed line is partitioned by the box with side length  $l_c$ . At time  $t_1$ , the wavepacket of the particle is shown by the

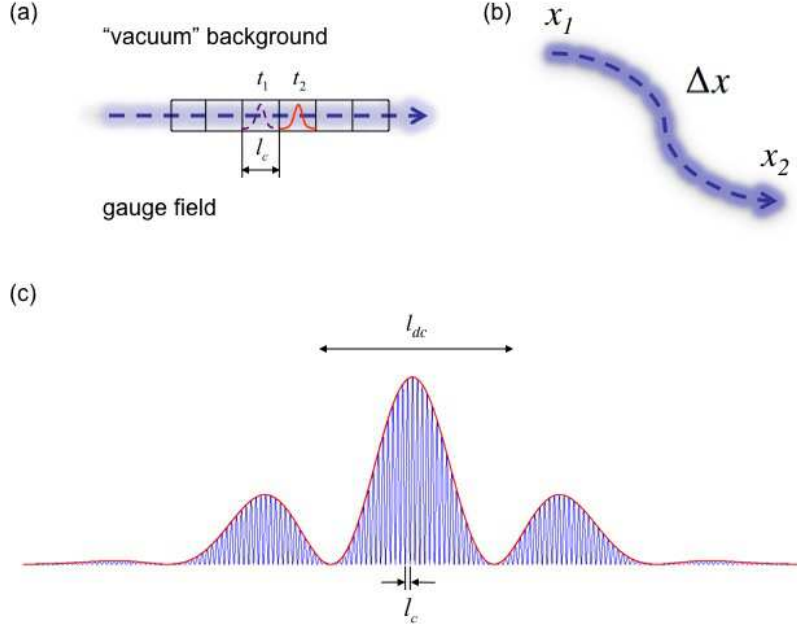


FIG. 1: Fig. (a) and Fig. (b) illustrate the physical origin to get the starting point given by Eq. (1). Fig. (c) shows the essential difference between  $l_c$  and  $l_{dc}$ .

dashed line. At a later time  $t_2$  ( $= t_1 + l_c/v$ ), the wavepacket of the particle is shown by the solid line. At time  $t_2$ , the information of the location of the particle at time  $t_1$  recorded by the “vacuum” background will be lost because of two reasons: (1) At time  $t_2$ , although the location of the particle with spatial resolution  $l_c$  is recorded by the “vacuum” background, the velocity information is highly uncertain. From  $\Delta x \Delta p \geq \hbar/2$ , the velocity uncertainty of the particle is  $\Delta v \sim c$ . In this situation, at time  $t_2$ , after the position of the particle is recorded by the “vacuum” background, the history of the particle is lost from the view of the “vacuum” background. (2) At time  $t_1$ , although the location with spatial resolution  $l_c$  is recorded by the “vacuum” background, it will be lost rapidly after a displacement at time  $t_2$ . At time  $t_1$ , the location record is due to the coupling between the particle and the “vacuum” background. Because the “vacuum” background is highly fluctuating, at time  $t_2$ , the location information at time  $t_1$  will be lost because of the re-establishment of the thermal equilibrium in the “vacuum” background.

The above analysis shows that there is an entropy increase of  $\Delta S \sim k_B$  after the dis-

placement of  $l_c$ . From this result, it is straightforward to get

$$\Delta S = \frac{\alpha}{2\pi} k_B \frac{mc}{\hbar} \Delta x, \quad (6)$$

with  $\Delta x$  being the overall length of the particle trajectory, irrelevant to the trajectory shape. By setting  $\alpha = 4\pi^2$ , we get Eq. (1). As shown in Fig. 1(b), for a trajectory with length  $\Delta x$ , the bits of information  $\sim \Delta x/l_c$ . However, from the view of “vacuum” background, all these information are lost at the final location of this trajectory. The linear relation between  $\Delta S$  and  $\Delta x$  is due to the fact that the overall information in units of bit is proportional to the overall length of a trajectory.

To justify further the above derivation, we give here several discussions.

(1) In the above derivation, we assume a strong coupling between the particle and the “vacuum” background, so that the location of the particle can be recorded by the “vacuum” background. As shown in Sec. IV, the temperature of the “vacuum” background is about  $T_v = 9.0 \times 10^{31}$  K. Thus, the “vacuum” background is in fact full of various gauge fields, virtual matter-antimatter pairs. This implies a strong coupling between the particle and the “vacuum” background.

(2) One should distinguish two sorts of coherence lengths: the coherence length  $l_c$  given by Eq. (5) and the ordinary thermal de Broglie wave length. In the non-relativistic approximation, the thermal de Broglie wave length reflects the spatial coherence length which is  $l_{de} \sim \hbar/\Delta p$ . We stress that  $l_c$  and  $l_{de}$  have different physical origins.  $l_c$  originates from the coupling between the particle and the “vacuum” background. Because of the extremely high temperature of the “vacuum” background and strong coupling, the special relativity and quantum mechanics are used to calculate  $l_c$ . In contrast to  $l_c$ ,  $l_{de}$  originates from the coupling or interaction between the particle we studied and other particles or environment. To clearly elaborate this, we take a hydrogen atom as an example, which consisting of an electron and a proton. For a system consisting of a large number of hydrogen atoms,  $l_{de} = \sqrt{2\pi\hbar}/\sqrt{mk_B T_{sys}}$  with  $T_{sys}$  being the ordinary temperature in thermal equilibrium.  $l_{de}$  physically originates from the interatomic interaction or the coupling with the environment. For Bose-Einstein condensation of atomic hydrogen [9],  $l_{de}$  can arrive at 300 Å. For the proton in a hydrogen, the coherence length of the proton is about the diameter of the hydrogen atom, *i. e.*  $l_{proton} \sim 1$  Å. This sort of  $l_{proton}$  can be obtained through the non-relativistic approximation. It is clear that  $l_{proton}$  physically originates from the electromagnetic in-

interaction between proton and electron in a same hydrogen. For the quark in the proton, the coherence length  $l_{quark}$  of the quark is about the radius of the proton (which is about  $8.7 \times 10^{-16}$  m). Because the proton is composed of three quarks, we approximate the quark mass (rather than the “naked” quarks) as one third of the proton mass. From Eq. (5), we have  $l_c = 3.96 \times 10^{-16}$  m for the quark. Quite interesting,  $l_{quark}$  agrees with this value of  $l_c$ . This is understandable, because the quark in the proton has strong coupling with the “vacuum” background due to strong interaction. This means that the coherence length of quarks should be always of the order of several  $10^{-16}$  m. For a free quark, its velocity uncertainty could be smaller than  $c$  in principle, and thus larger than  $l_c$ . Therefore, the existence of a free quark contradicts with the coherence length of  $l_c$ . This implies in a simple way the well-known quark confinement.

(3) Because  $l_{de}$ ,  $l_{proton}$  and  $l_{quark}$  have different physical origin, they have different physical significance. In particular, for the situations of  $l_{proton} \ll l_{de}$  and  $l_{quark} \ll l_{de}$ ,  $l_{de}$  still has physical significance for a whole atom. For sufficient low temperature, there is still Bose-Einstein condensation of dilute atomic gas, regarded of much smaller  $l_{proton}$  and  $l_{quark}$ . The physical significance of  $l_{de}$  is due to the fact that when interatomic interaction is addressed, the freedom of internal motion of proton and electron in an atom is frozen out. It is similar for  $l_{proton}$ , because the quark confinement means that the interaction between quark and “vacuum” background is screened. In a sense, the strong coupling between the quark and “vacuum” background is blurred when the coherence effect of the whole atom or proton is addressed. In Fig. 1(c), we show  $l_c$  ( $\sim l_{quark}$ ) and  $l_{de}$  for an atom. Nevertheless, we will show in due course that the presence of  $l_c$  and thus Eq. (1) will play important role in the derivation of gravity.

(4) Because of the “blurred” effect for  $l_c$  ( $\sim l_{quark}$ ), one expects that the gravity between two atoms is negligible, compared with other interactions. For example, it is clear that the formation of Bose-Einstein condensation of dilute atomic gases is completely irrelevant to the gravity interaction between atoms. Because the entropy is an extensive quantity, Eq. (1) can be directly generalized to an object comprising of a large number of fundamental particles. In this situation,  $\Delta S$  can be very large for sufficiently large mass of an object, and the gravity between different objects will play important role.

### III. A DERIVATION OF UNRUH EFFECT AND INERTIA LAW

The Unruh temperature [8] is the effective temperature experienced by a uniformly accelerating object in a vacuum field. Here we give a simple derivation of Unruh effect from Eq. (1). Considering a particle with acceleration  $a$ , we have  $x = at^2/2 + v_0t + x_0$ . Here  $v_0$  can be also regarded as the relative velocity of a reference system. From Eq. (6), we have

$$dS = \frac{\alpha k_B m c (at + v_0)}{2\pi\hbar} dt. \quad (7)$$

From Eq. (2), we have

$$dE \approx ma (at + v_0) dt. \quad (8)$$

Using  $dE = TdS$ , we get

$$k_B T = k_B \frac{dE}{dS} \approx \frac{2\pi\hbar a}{\alpha c}. \quad (9)$$

This gives the famous Unruh formula by setting  $\alpha = 4\pi^2$ .

From the formula

$$dE = Fdx = TdS, \quad (10)$$

we have

$$F = T \frac{dS}{dx}. \quad (11)$$

Using Eqs. (6) and (9), it is easy to get the following inertia law

$$F = ma. \quad (12)$$

### IV. A DERIVATION OF NEWTON'S LAW OF GRAVITATION

Following the Verlinde's idea, we give a derivation of Newton's law of gravitation. In our derivation, we will stress the local thermal equilibrium and the coherence length of the fluctuating "vacuum" background. We consider the emergent force between two objects with mass  $M$  and  $m$ , shown in Fig. 2(a). To solve this problem, we first consider the situation shown in Fig. 2(b), where the mass  $M$  is uniformly distributed in a region between two spherical surfaces with radius  $R$  and  $R + l_p$ . Assuming  $R \gg l_p$ , the overall volume of this region is  $V \approx 4\pi R^2 l_p$ . This region is divided into  $N = V/l_p^3 \approx 4\pi R^2/l_p^2$  equal parts. In this situation, in every part, the mass is  $m_0 = M/N = M l_p^2/4\pi R^2$ . Fig. 2(c) gives a cross section

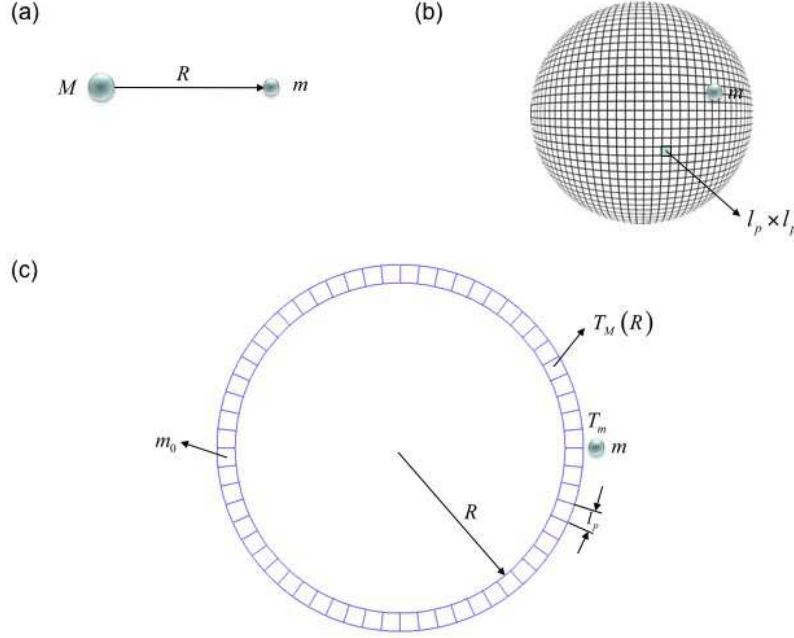


FIG. 2: The scheme to derive the Newton's law of gravity.

diagram of Fig. 2(b). We consider the emergent force with the “vacuum” background as the medium. More precisely, both objects have strong coupling with the “vacuum” background, and local thermal equilibrium will result in an emergent force.

Because the coupling with the “vacuum” background is considered, special relativity should be used from the beginning. From the mass-energy relation  $\varepsilon_0 = m_0 c^2$ , we get an effective temperature  $T_M$ , determined by

$$\frac{1}{2} k_B T_M = \varepsilon_0. \quad (13)$$

For a particle with  $m$  approaching this sphere, the Unruh temperature  $T_m$  of this particle should be equal to  $T_M$  because of thermal equilibrium, i.e.

$$T_M = T_m. \quad (14)$$

By using the Unruh formula (9), we get

$$a_m = \frac{M c^3 l_p^2}{R^2 \hbar}. \quad (15)$$

By introducing the gravitation constant  $G$ , we get  $l_p = \sqrt{\hbar G / c^3}$ , which is just the Planck length. We will discuss the origin of this Planck length in due course. In the above derivation, the Unruh formula is used, while Eq. (1) formula is not directly used.



There is another derivation of Eq. (15) by using Eq. (1). For the particle  $m$ , we have

$$dE = Fdx = T_m dS. \quad (16)$$

Using Eq. (14), we get

$$F = T_M \frac{dS}{dx}. \quad (17)$$

Using further Eq. (1), we get

$$\begin{aligned} F &= 2\pi k_B T_M \frac{mc}{\hbar} \\ &= \frac{GMm}{R^2}. \end{aligned} \quad (18)$$

From Eq. (15) (derived using Unruh's formula) and (18) (derived using Eq. (1)), we get the result of the inertia law  $F = ma_m$ . This is natural, because the Unruh formula can be derived from Eq. (1) and thermodynamical relation, while the inertia law can be derived from Eq. (1), thermodynamical relation and Unruh formula. Based on the consideration of central symmetry, we get further

$$\mathbf{F} = -\frac{GMm\mathbf{R}}{R^3}. \quad (19)$$

Compared with Verlinde's work, the merit of the above derivation lies in that the role of local thermal equilibrium is stressed in the derivation of the Newton's law of gravitation. Now we turn to consider an object of mass  $M$  locating at the position in Fig. 2 (a). From Gauss's flux theorem and Eq. (19), we have

$$\oint_S \mathbf{F} \cdot d\mathbf{A} = -GMm. \quad (20)$$

This shows that the gravitation is still given by Eq. (19) for this situation.

For a particle with mass  $M$ , from  $a_m = GM/R^2$  for another particle with mass  $m$  and the Unruh formula, there is an effective "vacuum" temperature due to the presence of  $M$ .

$$T_M = \frac{1}{2\pi} \frac{\hbar GM}{k_B c R^2}. \quad (21)$$

This leads to a physical picture that the presence of  $M$  establishes an effective temperature field distribution.

We assume that the object with mass  $M$  is uniformly distributed in the sphere with radius  $R_M$ . Eq. (21) can be used to calculate  $T_M$  for  $R > R_M$ . For  $R < R_M$ , however, using Eq. (20) and Unruh formula, the temperature is

$$T_M = \frac{1}{2\pi} \frac{\hbar GMR}{k_B c R_M^3}. \quad (22)$$

The overall temperature distribution is shown in Fig. 3. Therefore, the maximum temperature is

$$T_M^{\max} = \frac{1}{2\pi} \frac{\hbar G M}{k_B c R_M^2}. \quad (23)$$

From the mass-energy relation, even all the energy is transferred into temperature with only one freedom, we get a limit temperature

$$T_M^0 = \frac{2Mc^2}{k_B}. \quad (24)$$

Obvious, there is a request of

$$T_M^{\max} \leq T_M^0. \quad (25)$$

This means that

$$R_M \geq \sqrt{\frac{\hbar G}{4\pi c^3}} = \sqrt{\frac{1}{4\pi}} l_p. \quad (26)$$

This result shows that any object having strong coupling with the “vacuum” background cannot be distributed within a sphere of radius  $l_p/\sqrt{4\pi}$ , irrelevant to its mass. This provides a physical mechanism why in the previous calculation of  $T_M$  based on Eq. (13), the smallest region for the distribution of  $M$  is  $l_p^3$ .

We stress that even the “vacuum” background is completely continuous and smooth, there is still the coarse distribution of  $M$  due to gravitation, which originates from the coupling between matter and the “vacuum” background. Nevertheless, the above analysis gives a possible physical mechanism for the coarse graining of the space with the Planck length  $l_p$ . Here we consider further the physical significance of the Planck length  $l_p$ . The coarse graining of the “vacuum” background with length  $l_p$  means that the “vacuum” energy in the smallest space unit is

$$\varepsilon_u = \frac{\hbar c}{l_p}. \quad (27)$$

By using the mass-energy relation, the effective mass in this space unit is

$$m_u = \frac{\varepsilon_u}{c^2} = \frac{\hbar}{l_p c}. \quad (28)$$

The coherence length of the vacuum” background is then

$$l_c = \frac{2\pi\hbar}{m_u c} = 2\pi l_p. \quad (29)$$

We see that  $l_p$  gives also the coherence length of the “vacuum” fluctuations. This implies strongly again the coarse graining of space with the Planck length  $l_p$ .

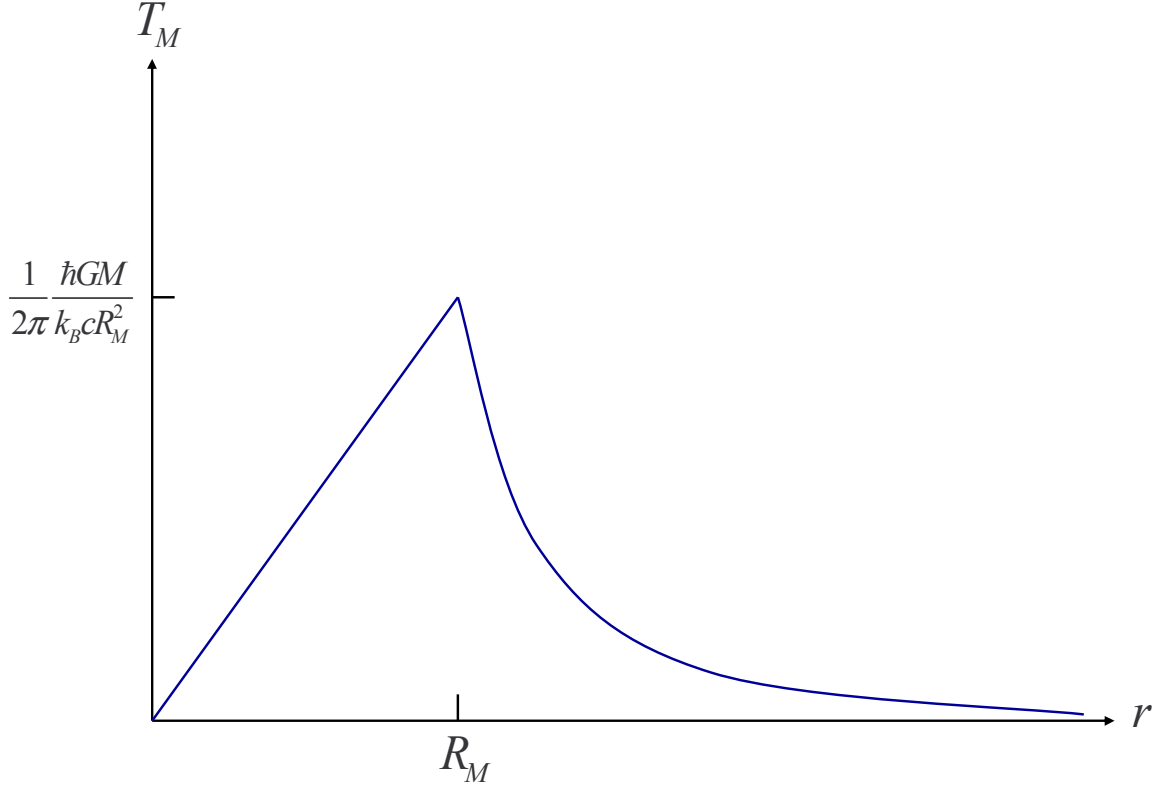


FIG. 3: Shown is the temperature distribution of an object uniformly distributed in a sphere with radius  $R_M$ .

One may argue that if there are “vacuum” fluctuations with a finite coherence length  $l_c$ , the increasing of  $l_c$  will lower the overall energy of the “vacuum” background, and thus is energy stable for infinite  $l_c$ . Based on the understanding of gauge theory, however, it is well-known that  $l_c$  should be finite and the “vacuum” energy density should be nonzero. Therefore, a more natural question is what length of  $l_p$  will make the “vacuum” background dynamically and thermodynamically stable. As shown above, the gravitation is no longer a fundamental force. It physically originates from the coupling between matter and “vacuum” background through gauge field. When the following expression is satisfied

$$\frac{Gm_u^2}{l_p} = m_u c^2, \quad (30)$$

it is understood that the “vacuum” background becomes stable. This gives

$$l_p = \frac{Gm_u}{c^2}. \quad (31)$$

Using further  $\varepsilon_u = \hbar c/l_p = m_u c^2$ , we get again  $l_c = \sqrt{\hbar G/c^3}$ .

From the above results, the temperature of the “vacuum” background is

$$T_v = \frac{2\hbar c}{k_B l_p}. \quad (32)$$

In this situation, we have  $T_v = 9.0 \times 10^{31}$  K. As emphasized above, this extremely high temperature is the thermal equilibrium due to the finite coherence length of highly fluctuating “vacuum” background and the gravity between these fluctuating “vacuum” background. This extremely high temperature of the “vacuum” background also means that the assumption of strong coupling between an object and the “vacuum” background in deriving the universal gravity is reasonable.

## V. THE WEAK EQUIVALENCE PRINCIPLE

It is well known that the equivalence principle is based on the assumption that the gravitational mass  $m_g$  is equal to the inertial mass  $m_I$ . It is still a mystery why  $m_g = m_I$  because it seems that the gravitational mass and inertial mass are completely different physical concept, if the universal gravitation is still regarded as a fundamental force. The above analysis has shown that Newton’s law of gravitation can be derived from the special relativity where the symbols  $M$  and  $m$  can both be regarded as the mass in the mass-energy relation. In all our derivations, we do not need especially introduce the gravitational mass at all. In this situation, the gravitational mass and inertial mass are in fact the same physical concept. Hence, the weak equivalence principle is also a derivable law from special relativity, quantum mechanics and true fundamental forces.

## VI. CONCLUSION AND DISCUSSION

The significance of excluding the gravity as a fundamental force has been pointed out in Refs. [1, 2]. In Fig. 4, we give the logical basis and the basic clues of our work to get Newton’s law of gravity, inertia law and weak equivalence principle. All our derivations depend on the special relativity, quantum mechanics and “vacuum” background. The information theory or thermodynamics is a tool to analysis the complex many-body system, similarly to the studies for a classical system with thermodynamics. Compared with Verlinde’s work, in the

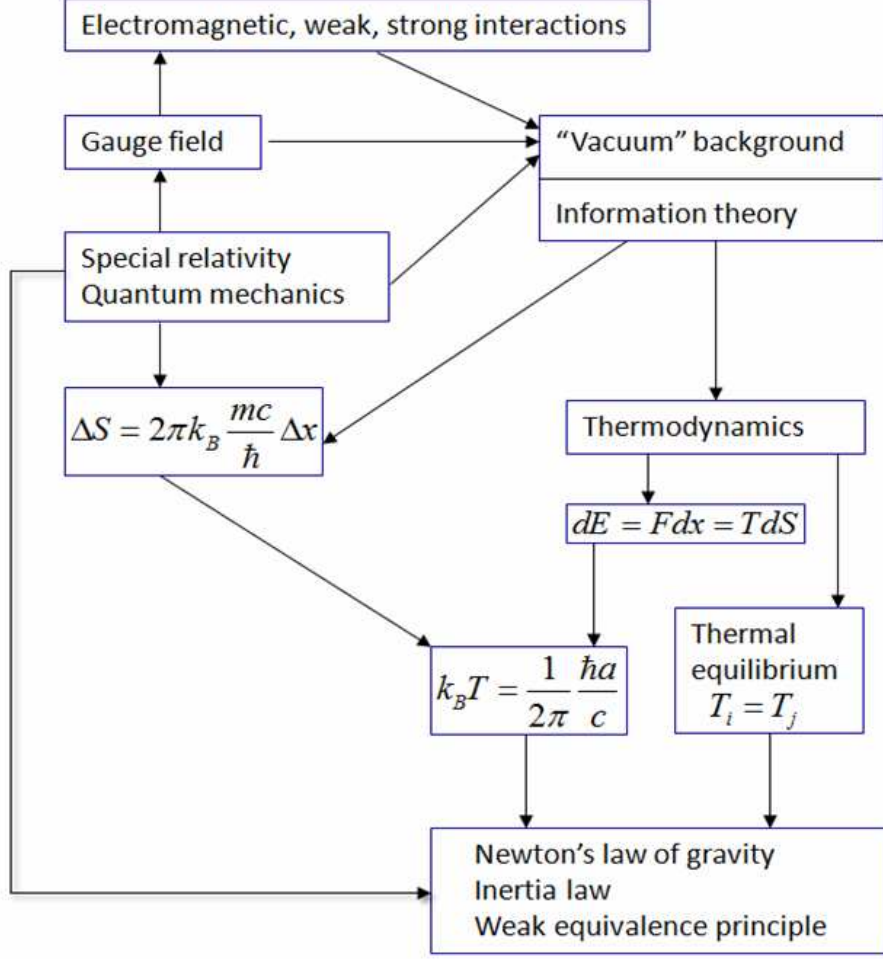


FIG. 4: Shown is the logical basis and the basic clues to get Newton's law of gravity, inertia law and weak equivalence principle.

present work, the concept of holographic screen is not absolutely necessary to derive the universal gravity and the laws of Newton.

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